Chapter 4

Forwards, futures and swaps
- Hợp đồng kỳ hạn, hợp đồng tương lai và hoán đổi

Chapter outlines – ý chính của chương

1. Determine forward/future price (Xác định giá kỳ hạn/TL)
2. Value forward contracts (Định giá hợp đồng kỳ hạn/hợp đồng tương lai)
3. Swap (Hoán đổi trong tài chính)
4. Value swaps (Định giá hoán đổi)

Determination of Forward and Futures Prices
Xác định giá kỳ hạn và tương lai

Read more in John Hull's book, chapter 5.

Short Selling (1 of 2)

- Short selling involves selling securities you do not own.
- Your broker borrows the securities from another client and sells them in the market in the usual way.

Consumption vs Investment Assets

- Investment assets are assets held by significant numbers of people purely for investment purposes (examples: gold, silver)
- Consumption assets are assets held primarily for consumption (examples: copper, oil)

Short Selling (2 of 2)

- At some stage, you must buy the securities so they can be replaced in the account of the client.
- You must pay dividends and other benefits the owner of the securities receives.
- There may be a small fee for borrowing the securities.

Example

- You short 100 shares when the price is $100 and close out the short position three months later when the price is $90.
- During the three months, a dividend of $3 per share is paid.
- What is your profit?
- What would be your loss if you had bought 100 shares?

Notation for Valuing Futures and Forward Contracts

S_0 : Spot price today

F_0 : Futures or forward price today

T : Time until delivery date

r : Risk-free interest rate for maturity T

An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a non-dividend-paying stock is $40.
  - The 3-month forward price is $43.
  - The 3-month US$ interest rate is 5% per annum.
- Is there an arbitrage opportunity (ignoring transaction costs)?

Yes!

If we borrow $40, buy a share and sell with forward price, we get $43.

Pay the interest for 3 months: 5%/ 4 x 40 = 0.5 ($)

After 3 months, we obtain how much per share?

An Arbitrage Opportunity?

- Suppose that:
  - The spot price of a non-dividend-paying stock is $40.
  - The 3-month forward price is $43.
  - The 3-month US$ interest rate is 5% per annum.
- Is there an arbitrage opportunity (ignoring transaction costs)?

Another Arbitrage Opportunity?

- Suppose that:
  - The spot price of non-dividend-paying stock is $40.
  - The 3-month forward price is US$39.
  - The 1-year US$ interest rate is 5% per annum (continuously compounded).
- Is there an arbitrage opportunity (ignoring other costs)?

Another Arbitrage Opportunity?

- Suppose that:
  - The spot price of non-dividend-paying stock is $40.
  - The 3-month forward price is US$39.
  - The 1-year US$ interest rate is 5% per annum (continuously compounded).
- Is there an arbitrage opportunity (ignoring other costs)?
- Yes, borrow a share, short sell to get $40. Put $40 to the bank to get 5% interest. In the same time we set up a forward contract to buy the share back in 3 months, we only need to pay $39 after 3 months to get the share and close the short position.
- Calculate the profit in this case?

The Forward Price (Equation 5.1)

If the spot price of an investment asset that provides no income is S_0 and the futures price for a contract deliverable in T years is F_0, then

$$F_0 = S_0 e^{rT}$$

where r is the T-year risk-free rate of interest.

In our examples, $S_0 = 40$, $T = 0.25$, and $r = 0.05$ so that

$$F_0 = 40e^{0.05\times 0.25} = 40.50$$

If Short Sales Are Not Possible…

Formula still works for an investment asset because investors who hold the asset will sell it and buy forward contracts when the forward price is too low.

When an Investment Asset Provides a Known Yield (Equation 5.3)

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

When an Investment Asset Provides a Known Income (Equation 5.2)

$$F_0 = (S_0 - \mathrm{I})e^{rT}$$

where I is the present value of the income during life of forward contract

Forward vs Futures Prices

- When the maturity and asset price are the same, forward and futures prices are usually assumed to be equal. (Eurodollar futures are an exception)
- In theory, when interest rates are uncertain, they are slightly different:
  - A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
  - A strong negative correlation implies the reverse

Stock Index (Equation 5.8)

- Can be viewed as an investment asset paying a dividend yield.
- The futures price and spot price relationship is therefore,

$$F_0 = S_0 e^{(r-q)\mathrm{T}}$$

where q is the average dividend yield on the portfolio represented by the index during life of contract.

Stock Index (continued)

- For the formula to be true, it is important that the index represent an investment asset.
- In other words, changes in the index must correspond to changes in the value of a tradable portfolio.
- Thze Nikkei index viewed as a dollar number does not represent an investment asset (See Business Snapshot 5.3)

Index Arbitrage (1 of 2)

- When $F_0 > S_0 e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures.

- When $F_0 < S_0 e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index.

Index Arbitrage (2 of 2)

- Index arbitrage involves simultaneous trades in futures and many different stocks.
- Very often a computer is used to generate the trades.
- Occasionally simultaneous trades are not possible and the theoretical no-arbitrage relationship between

  F_0 and S_0 does not hold (see Business Snapshot 5.4)

Futures and Forwards on Currencies (Equation 5.9)

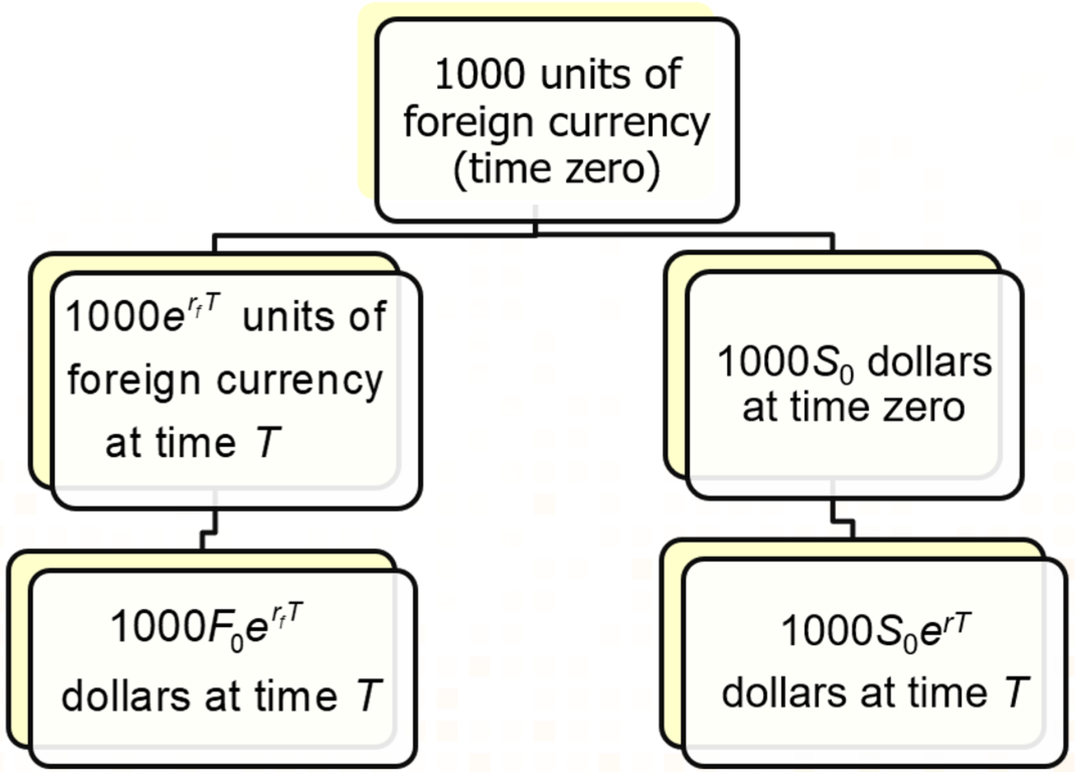
- A foreign currency is analogous to a security providing a yield (tương tự như một chứng khoán có sinh lãi r_f)
- The yield is the foreign risk-free interest rate.

- It follows that r_f is the foreign risk-free interest rate.

$$F_0 = S_0 e^{(r - r_f)T}$$

Explanation of the Relationship Between Spot and Forward

(Figure 5.1)



Consumption Assets: Storage is Negative Income (Equations 5.11 and 5.12)

$$F_0 \leq S_0 e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.

Alternatively,

$$F_0 \leq (S_0 + U)e^{rT}$$

where U is the present value of the storage costs.

The Cost of Carry (Equation 5.19)

- The cost of carry, *c*, is the storage cost plus the interest costs less the income earned
- For an investment asset $F_0 = S_0 e^{cT}$
- For a consumption asset $F_0 \leq S_0 e^{cT}$
- The convenience yield on the consumption asset, *y*, is defined so that

$$F_0 = S_0 e^{(c-y)T}$$

Futures Prices & Expected Future Spot Prices (Equation 5.20)

- Suppose *k* is the expected return required by investors in an asset.
- We can invest $F_0 e^{-rT}$ at the risk-free rate and enter into a long futures contract to create a cash inflow of S_T at maturity.
- This shows that

$$F_0 e^{-rT} e^{kT} = E(S_T)$$

or

$$F_0 = E(S_T) e^{(r-k)T}$$

Futures Prices & Future Spot Prices (continued)

No Systematic Risk	$k = r$	$F_0 = E(S_T)$
Positive Systematic Risk	$k > r$	$F_0 < E(S_T)$
Negative Systematic Risk	$k < r$	$F_0 > E(S_T)$

Positive systematic risk: stock indices
Negative systematic risk: gold (at least for some periods)

Valuing a Forward Contract

- A forward contract is worth zero (except for bid-offer spread effects) when it is first negotiated.
- Later, it may have a positive or negative value.
- Suppose that K is the delivery price and F_0 is the forward price for a contract that would be negotiated today.

Valuing a Forward Contract (Equation 5.4)

- By considering the difference between a contract with delivery price K and a contract with delivery price

  F_0 we can deduce that:

  - the value of a long forward contract is

$$(F_0 - K)e^{-rT}$$

  - the value of a short forward contract is

$$(K - F_0)e^{-rT}$$

Valuing the forward contract

- A long forward contract on zero-coupon bond was established some months ago. There is 9 months from now to the maturity of the forward contract, which happens before the maturity of the bond. The bond spot price is $48, the delivery price in forward contract is $48.5, the 9-month risk-free rate is 5% compounded continuously. Value the above forward contract?

Solution.

The forward price in 9 months (0.75 year) should be

$$F_0 = 48 \times e^{0.05 \times 0.75} \approx 49.8342 \text{ (\$)}.$$

The value of the forward contract approximates

$$f = (49.8342 - 48.5) \times e^{-0.05 \times 0.75} \approx 1.2851 \text{ (\$)}.$$

Excercises

- Chapter 5 in Hull’s book.

Swaps – hoán đổi

Nature of Swaps

A swap is an agreement to exchange cash flows at specified future times according to certain specified rules.

An Example of a “Plain Vanilla” Overnight Indexed Swap

- The deal entered into on March 8, 2022, where Apple agrees to receive 3-month SOFR and pay a fixed rate of 3% per annum every 3 months for 2 years on a notional principal of $100 million.
- Next slide illustrates cash flows that could occur (Day count conventions are not considered).

Cash Flows to Apple for One Outcome (Table 7.1)

Date	*SOFR Rate (%)*	*Floating Received ('000s)*	*Fixed Paid ('000s)*	*Net cash flow ('000s)*
June 8, 2022	2.20	550	750	–200
Sept 8, 2022	2.60	650	750	–100
Dec. 8, 2022	2.80	700	750	–50
Mar. 8, 2023	3.10	775	750	+25
June 8, 2023	3.30	825	750	+75
Sept 8, 2023	3.40	850	750	+100
Dec 8, 2023	3.60	900	750	+150
Mar 8, 2024	3.80	950	750	+200

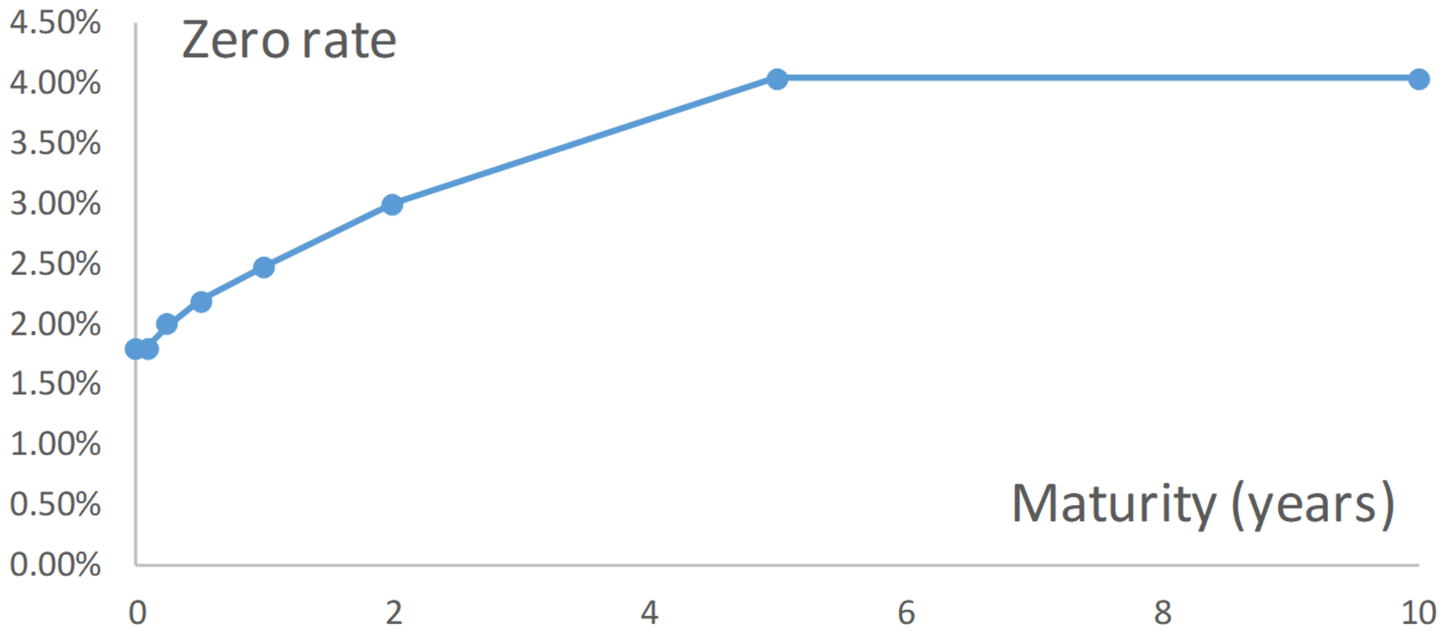
Determination of Risk-Free Interest Rates

- OIS rates out to one year define zero rates because they typically involve a single exchange.
- OIS rate for contracts lasting longer than one year define par yield.
- The bootstrap method can be used to determine the zero curve.

Bootstrap Example (Table 7.3)

OIS Maturity	*OIS Rate*	*Compound. Freq. for OIS rate*	*Zero rate (cont comp.)*
1 month	1.8%	Monthly	1.7987%
3 months	2.0%	Quarterly	1.9950%
6 months	2.2%	Semiannually	2.1880%
12 month	2.5%	Annually	2.4693%
2 years	3.0%	Quarterly	2.9994%
5 years	4.0%	Quarterly	4.0401%

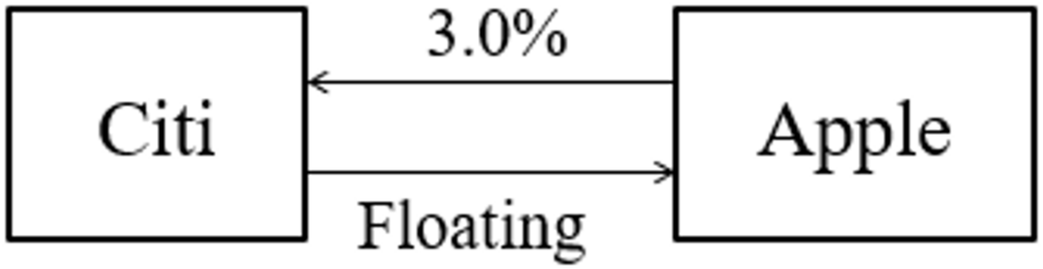
Zero Rate Given by Bootstrap Method (Figure 7.2)



Typical Uses of an Interest Rate Swap

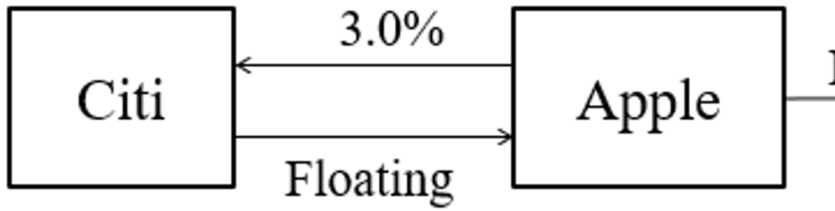
- Converting a liability from:
  - fixed rate to floating rate
  - floating rate to fixed rate
- Converting an investment from:
  - fixed rate to floating rate
  - floating rate to fixed rate

OIS Between Apple and Citigroup (Figure 7.1)

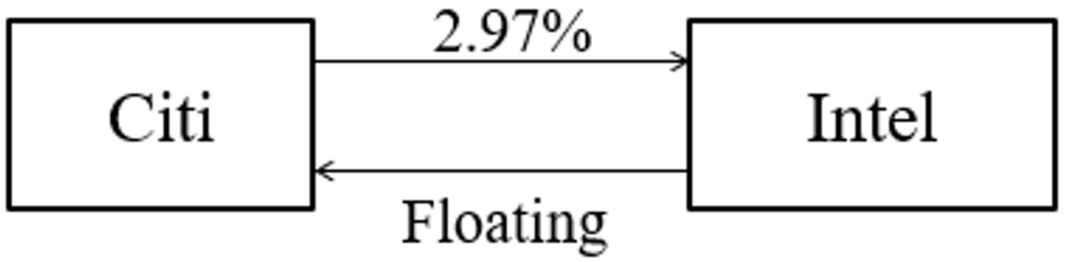


Apple Transforms a Liability from Floating to Fixed

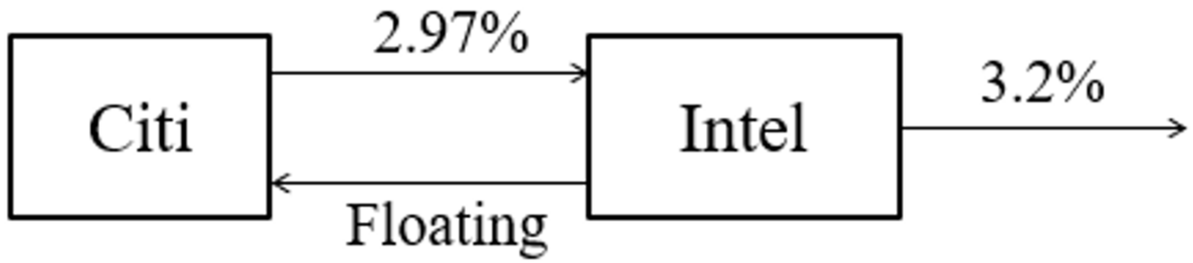
(Figure 7.3)



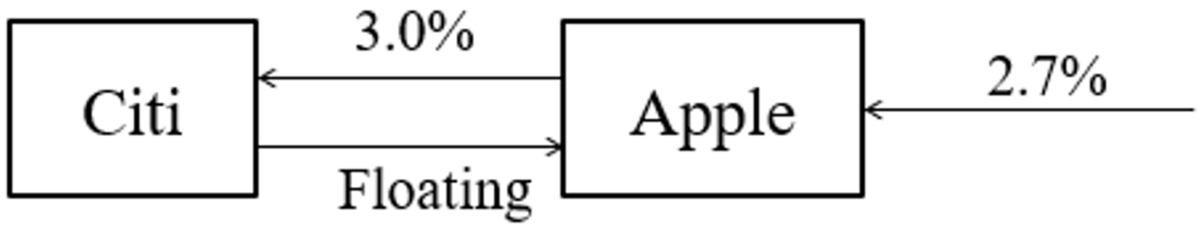
Interest Rate Swap Between Citigroup and Intel



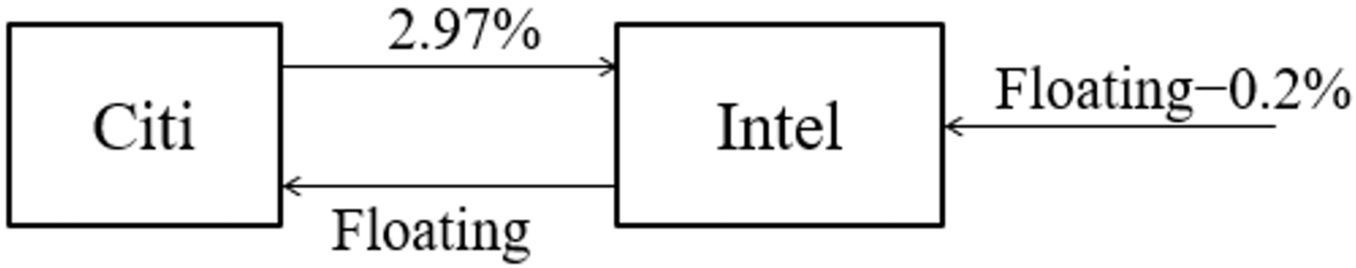
Intel Transforms a Liability from Fixed to Floating (Figure 7.4)



Apple Transforms an Asset from Fixed to Floating (Figure 7.5)



Intel Transforms an Asset from Floating to Fixed (Figure 7.6)



Quotes By a Swap Market Maker (Table 7.4)

Maturity	*Bid (%)*	*Ask (%)*	*Swap Rate (%)*
2 years	2.97	3.00	2.985
3 years	3.05	3.08	3.065
4 years	3.15	3.19	3.170
5 years	3.26	3.30	3.280
7 years	3.40	3.44	3.420
10 years	3.48	3.52	3.500

Day Count

- A day count convention is specified for fixed and floating payments.
- For example, SOFR is likely to be actual/360 in the U.S.
- The fixed rate might be quoted with actual/365 or 30/360.

Confirmations

- Confirmations specify the terms of a transaction.
- The International Swaps and Derivatives has developed Master Agreements that can be used to cover all agreements between two counterparties.
- CCPs are used for standard swaps between two financial institutions.

The Comparative Advantage Argument (Table 7.5)

- AAACorp wants to borrow floating
- BBBCorp wants to borrow fixed

	Fixed	*Floating*
AAACorp	4.00%	Floating ref − 0.1%
BBBCorp	5.20%	Floating ref + 0.6%

Example

AAA Corporation and BBB Corporation are offered some rates as the below table.

	Fixed	Floating
AAA	4.0%	-0.1% + 6-month LIBOR
BBB	5.2%	0.6% + 6-month LIBOR

AAA Corporation wants to borrow floating rate less than the above offer (6-month LIBOR - 0.1%). BBB Corporation wants to borrow fixed rate less than the offer at 5.2%.

a. How to design a suitable direct trading swap so that both sides get what they want?
b. In case the two sides make the swap via a financial institution and the financial institution charge each side 2 basis points (0.02%) then how the swap would be?

Solution.

We want to calculate the fixed rate r so that both side gain equally.
Note that the fixed interest difference

$$5.2\% - 4\% = 1.2\%.$$

The difference in two floating interests

$$\text{LIBOR} + 0.6\% - (\text{LIBOR} - 0.1\%) = 0.7\%$$

Using interest swap, the interest gains for each side should be

$$(1.2\% - 0.7\%)/2 = 0.25\%.$$

So, one could design an interest swap such that AAA Corporation could borrow with a better floating rate

$$\text{LIBOR} - 0.1\% - 0.25\% = \text{LIBOR} - 0.35\%$$

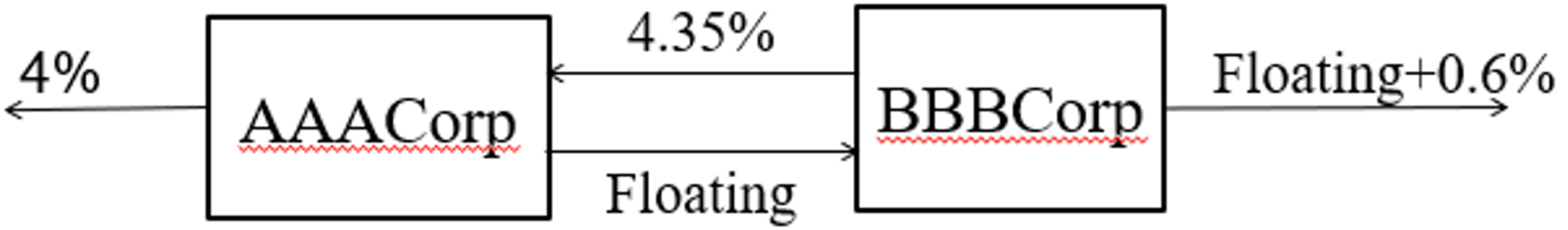
and BBB Corporation could borrow better fixed rate

$$5.2\% - 0.25\% = 4.95\%.$$

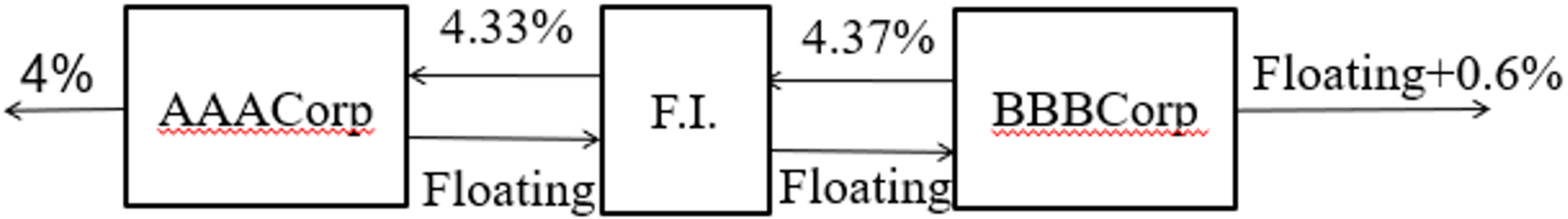
The new floating rate for AAA Corporation

$$\text{LIBOR} + 4\% - r = \text{LIBOR} - 0.35\% \text{ so } r = 4.35\%.$$

A Swap where Companies Trade Directly with Each Other (Figure 7.7)



The Swap When a Financial Institution (F.I.) Is Involved (Figure 7.8)



Criticism of the Comparative Advantage Argument

- The 4.0% and 5.2% rates available to AAACorp and BBBCorp in fixed rate markets are 5-year rates.
- The rates available in the floating rate market are 3-month rates.
- BBBCorp's fixed rate depends on the spread above floating it borrows at in the future.

Valuation of an Interest Rate Swap

- Initially, interest rate swaps are worth close to zero.
- At later times, they can be valued as a portfolio of forward rate agreements (FRAs).
- The procedure is to:
  - Calculate floating forward rates
  - Calculate the swap cash flows that will occur if floating forward rates are realized
  - Discount these swap cash flows at OIS rates

Valuation of an Interest Rate Swap

- Ban đầu, hoán đổi có giá trị 0. Một thời gian sau, nó có thể có giá trị khác 0 (được định giá tương tự một danh mục các hợp đồng lãi suất kỳ hạn – FRAs).
- Các bước tiến hành định giá:
  - Xác định lãi suất kỳ hạn thả nổi
  - Tính toán dòng tiền hoán đổi
  - Chiết khấu dòng tiền thu được theo lãi suất OIS

Example 7.1 (1 of 2)

- Swap involves paying 3% per annum and receiving SOFR every six months on $100 million.
- Swap has 1.2 years remaining (exchanges in 0.2, 0.7, and 1.2 years).
- Risk-free rate for 0.2, 0.7, and 1.2 years are 2.8%, 3.2% and 3.4%, respectively (continuously compounded).
- Rate observed for last 0.3 years is 2.3% continuously compounded.

Example 7.1 (2 of 2)

- Floating rate for the exchange at 0.2 years is assumed to be

  $0.6 \times 2.3\% + 0.4 \times 2.8\%$ or 2.50% (cont. comp.) or 2.516% (sa)

- Forward rate for 0.2 to 0.7 years is 3.36% (cont. comp.) or 3.388% (sa)
- Forward rate for 0.7 to 1.2 years is 3.68% (cont. comp.) or 3.714% (sa)

Calculations – tính toán ($ million)

Time (yrs)	Fixed cash flow	Floating cash flow	Net cash flow	Discount factor	PV of net cash flow
0.2	–1.5000	+1.258	–0.241	0.9944	–0.242
0.7	–1.5000	+1.694	+0.194	0.9778	+0.190
1.2	–1.5000	+1.857	+0.357	0.9600	+0.343
					+0.292

Value of swap is $0.292 million.

Value Changes Through Time

- To party paying fixed:
  - How is swap value expected to change through time when term structure is upward sloping? (See more at figure 7.9, page 186)
  - How is swap value expected to change through time when term structure is downward sloping?

An Example of a Fixed-for-Fixed Currency Swap (Figure 7.10)

Five-year agreement by BP to:

- Pay 3% on a US dollar principal of $15,000,000
- Receive 4% on a sterling principal of £10,000,000

Exchange of Principal

- In an interest rate swap, the principal is not exchanged.
- In a currency swap, the principal is exchanged at the beginning and the end of the swap.

The Cash Flows (Table 7.6)

Date	Dollar Cash Flows (millions)	Sterling cash flow (millions)
Feb 1, 2022	+15.00	–10.00
Feb 1, 2023	–0.45	+0.40
Feb 1, 2024	–0.45	+0.40
Feb 1, 2025	–0.45	+0.40
Feb 1, 2026	–0.45	+0.40
Feb 1, 2027	–15.45	+10.40

Typical Uses of a Currency Swap

- Conversion from a liability in one currency to a liability in another currency.
- Conversion from an investment in one currency to an investment in another currency.

Comparative Advantage May Be Real Because of Taxes

- General Electric wants to borrow AUD.
- Quantas wants to borrow USD.

Borrowing costs after adjusting for the differential impact of taxes could be:

	USD	AUD
General Electric	5.0%	7.6%
Quantas	7.0%	8.0%

Valuation of Fixed-for-Fixed Currency Swaps

Fixed for fixed currency swaps can be valued either using forward rates or as the difference between 2 bonds.

Valuation of other Currency Swaps

- Fixed-for-floating: equivalent to a fixed-for-fixed currency swap plus a fixed for floating interest rate swap
- Floating-for-floating: equivalent to a fixed-for-fixed currency swap plus two floating interest rate swaps

Currency swap example (fixed-fixed)

- All Japanese interest rates are 1.5% per annum (cont. comp.).
- All USD interest rates are 2.5% per annum (cont. comp.).
- 3% is received in yen; 4% is paid in dollars. Payments are made annually.
- Principals are $10 million and 1,200 million yen.
- Swap will last for 3 more years.
- Current exchange rate is 110 yen per dollar.

Hoán đổi tiền tệ (lãi suất cố định)

- Tất cả các lãi suất của yên Nhật Bản là 1,5% mỗi năm (gộp liên tục).
- Tất cả các lãi suất USD đều là 2,5% một năm (gộp liên tục).
- Nhận 3% bằng đồng yên; trả 4% bằng đô la. Thanh toán được thực hiện hàng năm.
- Số tiền hoán đổi ban đầu là 10 triệu đô la và 1200 triệu yên.
- Hoán đổi sẽ kéo dài thêm 3 năm nữa.
- Tỷ giá hối đoái hiện tại là 110 yên một đô la.

Ref p173 (199) Ver. 10.

Valuation in Terms of Forward exchange Rates – Định giá dùng tỷ giá trao đổi kỳ hạn (Exp 7.2 **Ver 11, 7.3 in Ver 10, p.172(198),** chú ý CT tỷ giá **(5.9) p.121(146)**)

Time	*Dollar Cash Flow*	*Yen cash flow*	*Forward exchange rate (1 Yên trong TL bằng ?$)*	*Dollar value of yen cash flow*	*Net cash flow*	*Present value (dùng chiết khấu theo $)*
1	–0.4	+36	**0.009182**	0.3306	–0.0694	–0.0677
2	–0.4	+36	**0.009275**	0.3339	–0.0661	–0.0629
3	–10.4	+1236	**0.009368**	11.5786	+1.1786	+1.0934
Total						+0.9629

Valuation in Terms of Bonds- Định giá theo trái phiếu

(Vd 7.3, ($)r=0.025, r_F=0.015 gộp l. t.)

Time *Thời gian*	*Cash Flows ($ millions)* *Dòng tiền*	*PV* *($ millions)* *Giá trị HT*	*Cash flows (millions of yen)*	*PV (millions of yen)*
1	0.4	0.3901	36	35.46
2	0.4	0.3805	36	34.94
3	10.4	9.6485	1,236	1,181.61
Total		10.4191		1,252.01

$$\text{Value} = 1{,}252.01/110 - 10.4191 = +0.9629$$ millions of dollars

Exercises – Bài tập

A financial institution has a currency swap for some years ago with principals are $120 million and 14 000 million yen. Once per year they pay 3.5% in dollars and receive 3% in yen. All Japanese interest rates are 1.2% per annum and USD interest rates are 2.2% per annum, compounded continuously. The swap would last for another 3 years. The current exchange rate is 105 yen for 1 dollar, compute the current value of the swap?

Một tổ chức tài chính đã hoán đổi tiền tệ cách đây vài năm với số tiền gốc là 120 triệu đô la và 14 000 triệu yên. Mỗi năm một lần họ trả 3,5% bằng đô la và nhận 3% bằng yên. Tất cả lãi suất của yên Nhật là 1,2% / năm và lãi suất USD là 2,2% / năm, được cộng gộp liên tục. Việc hoán đổi sẽ kéo dài thêm 3 năm nữa. Tỷ giá hối đoái hiện tại là 105 yên đổi 1 đô la, hãy tính giá trị hiện tại của giao dịch hoán đổi?

THANK YOU

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ĐẠI HỌC QUỐC GIA THÀNH PHỐ HỒ CHÍ MINH
TRƯỜNG ĐẠI HỌC KINH TẾ - LUẬT

Số 669 Quốc lộ 1, Khu phố 3, Phường Linh Xuân,
TP. Thủ Đức, TP. Hồ Chí Minh.